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CHAPTER 1

Numbers

If you look at numbers around you, you may observe that besides positive numbers, negative numbers are also used. We use such numbers to represent values less than zero in many different situations. For example, we come across negative numbers in weather reports and on food packaging. The temperature –3 °C means 3 degrees below zero.

Many of us will recognise the negative value in stored value card as shown by the Add Value Machine (AVM) screen reader above. Such number is identified by a negative sign (–) appearing before it.

Look around for more examples where numbers are being used to represent values less than zero.

In this chapter, you will learn to:

- find primes and perform prime factorisation,
- find the Highest Common Factor (HCF) and the Lowest Common Multiple (LCM) by prime factorisation,
- perform four operations on negative numbers and integers,
- find squares, cubes, square roots and cube roots by prime factorisation,
- perform four operations on rational numbers and real numbers,
- perform calculations using a calculator,
- represent and order numbers on a number line,
- use mathematical symbols $<$, $>$, $\leqslant$ and $\geqslant$.

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CHAPTER 1

Numbers

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EXAMPLE 3

Find the prime factors of 600, giving your answer in index notation.

SOLUTION

Using the 'continuous division' method

2	600
2	300
2	150
3	75
5	25
5	5
	1

$600 = 2 \times 2 \times 2 \times 3 \times 5 \times 5$

$= 2^3 \times 3 \times 5^2$ (index notation)

Go to Workbook Exercise 1, page 12

Practice 1A

Basic

1. List all the prime numbers less than 20.

2. Give a list of the following sets of numbers.
   (a) The next four prime numbers greater than 47.
   (b) The first five prime numbers between 100 and 200.

3. Given three prime numbers 13, 37 and 43, check whether:
   (a) the sum of these prime numbers is a prime number,
   (b) the product of the lowest and highest prime numbers is a prime number.

4. Use the 'factor tree' method to find the prime factors of 114.

5. Find the prime factors of each of the following numbers, giving your answers in index notation.
   (a) 294 (b) 1980 (c) 5695 (d) 17 325

1.2 Highest Common Factor (HCF) and Lowest Common Multiple (LCM)

Highest Common Factor (HCF)

A common factor for two or more numbers is a factor that is common to all these numbers. There may be more than one common factor.

$819 = 3 \times 3 \times 7 \times 13$

$1890 = 2 \times 3 \times 5 \times 7 \times 9$

3 and 7 are common factors of 819 and 1890.

The **highest common factor** (HCF) of two or more numbers is the largest common factor of these numbers.

Consider the numbers 16 and 24.
The factors of 16 are 1, 2, 4, 8 and 16.
The factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.

We see that 2, 4 and 8 are the common factors of 16 and 24.
And 8 being the largest common factor of 16 and 24, we say that the highest common factor (HCF) of 16 and 24 is 8.

We may use prime factorisation to find the HCF of these numbers.

$16 = 2 \times 2 \times 2 \times 2 = \boxed{2^4} \times \boxed{1}$

$24 = 2 \times 2 \times 2 \times 3 = \boxed{2^3} \times \boxed{3}$

2^3 1

Choose the smaller number from each set of boxed numbers and multiply them.

Therefore, the HCF of 16 and 24 is $2^3 \times 1 = 2^3$ or 8.

Caution

Both 16 and 24 are divisible by 2^3 but only 16 is divisible by 2^4.

For a number to be a common factor of a set of numbers, all the numbers in this set must be divisible by it.

Alternatively, by using continuous division,

2	16	24
2	8	12
2	4	6
	2	3

We stop here as both 2 and 3 have no common factor except 1.

Multiply these numbers.

HCF $= 2 \times 2 \times 2 = 8$

Thus, the HCF of 16 and 24 is 8.

Let's Explore

The famous Greek mathematician Euclid invented an unusual way of finding the HCF of **two** whole numbers.

Listed below is the method that he used for finding the HCF of 12 and 20.

- Write the two numbers 12 and 20 in descending order in a row.

  20, 12

- Find their difference (i.e. 8) and add it to the row.

  20, 12, 8

- Find the difference between the last two numbers and write this new difference at the end of the row.

  20, 12, 8, 4

- Continue finding the difference between the last two numbers until you reach zero.

  20, 12, 8, 4, 4

  20, 12, 8, 4, 4, 0

- The HCF is the number before zero.

  ∴ The HCF of 20 and 12 is 4.

Test out the method using any other numbers. Does it work all the time?

Do you think the process works for finding the HCF of three whole numbers?

EXAMPLE 4

Find the HCF of 12, 36 and 48.

SOLUTION

$$\begin{aligned} 12 &= 2 \times 2 \times 3 &&= \boxed{2^2} \times \boxed{3} \\ 36 &= 2 \times 2 \times 3 \times 3 &&= \boxed{2^2} \times \boxed{3^2} \\ 48 &= 2 \times 2 \times 2 \times 2 \times 3 &&= \boxed{2^4} \times \boxed{3} \end{aligned}$$

$$\underbrace{2^2 \quad 3}$$

Select the smallest number from each set of boxed numbers and multiply them.

Thus, the HCF of 12, 36 and 48 $= 2^2 \times 3$

$= 12$

Alternatively, by using continuous division,

2	12	36	48
2	6	18	24
3	3	9	12
	1	3	4

Multiply these numbers.

HCF $= 2 \times 2 \times 3 = 12$

Thus, the HCF of 12, 36 and 48 is 12.

Lowest Common Multiple (LCM)

A common multiple of two or more numbers is a product of the factors for these numbers.

The **Lowest Common Multiple** (LCM) of two or more numbers is the smallest common multiple of these numbers.

Consider the numbers 16 and 24.

Multiples of 16 are 16, 32, 48, 64, 80, 96, 112, 128, 144, …

Multiples of 24 are 24, 48, 72, 96, 120, 144, 168, …

The common multiples of 16 and 24 are 48, 96, 144, …

Since 48 is the smallest among these common multiples, we say the LCM of 16 and 24 is 48.

We can also find the LCM of 16 and 24 through prime factorisation.

$16 = \boxed{2^4}$

$24 = \boxed{2^3} \times 3$

Choose the larger number in this set, i.e. 2^4 and multiply it by 3.

Thus, the LCM of 16 and 24 is $2^4 \times 3 = 48$.

Caution

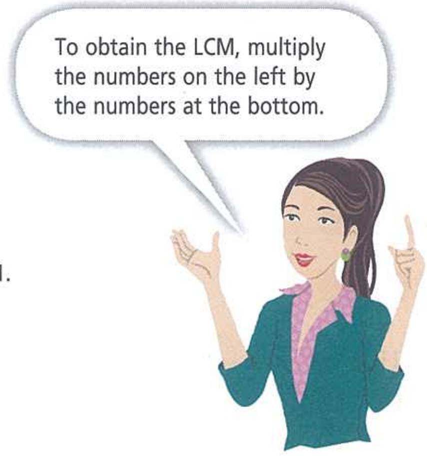
The value of $2^4 \times 3$ is divisible by both 16 and 24.

For a number to be a common multiple of a set of numbers, this number must be divisible by all the numbers in the set.

Alternatively,

2	16	24
2	8	12
2	4	6
	2	3

We stop dividing when there are no common factors except 1.



LCM $= 2 \times 2 \times 2 \times 2 \times 3$

$= 48$

EXAMPLE 5

Find the LCM of 90, 135 and 150.

SOLUTION

By prime factorisation,

$$\begin{aligned} 90 &= \boxed{2} \times \boxed{3^2} \times \boxed{5} \\ 135 &= \boxed{1} \times \boxed{3^3} \times \boxed{5} \\ 150 &= \boxed{2} \times \boxed{3} \times \boxed{5^2} \end{aligned}$$

$$2 \qquad 3^3 \qquad 5^2$$

Thus, the LCM is $2 \times 3^3 \times 5^2 = 1350$.

Alternatively, we have

5	90	135	150
3	18	27	30
3	6	9	10
2	2	3	10
	1	3	5

Carry 10 to the next line and divide 6 and 9 by 3.

Carry 3 to the next line and divide 2 and 10 by 2. We stop dividing when any two of the numbers have no common factors except 1.

Thus, the LCM is $5 \times 3 \times 3 \times 2 \times 3 \times 5 = 1350$.

Go to Workbook Activity 2, page 3
Go to Workbook Project A, page 26
Go to Workbook Exercise 2, page 13

Practice 1B

Basic

1. Find the HCF of each set of numbers below.
   (a) 27, 45 (b) 56, 70 (c) 36, 90, 126 (d) 50, 75, 115

2. Find the LCM of each of the following sets.
   (a) 9, 11, 99 (b) 30, 24, 120 (c) 16, 28, 44 (d) 39, 45, 54

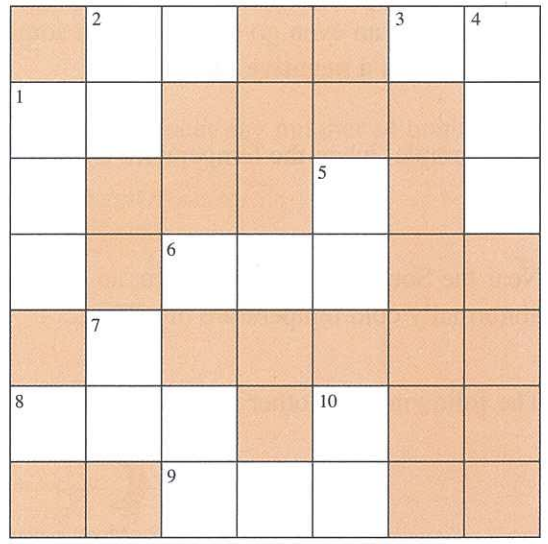
3. Find the HCF and LCM of each of the following sets.
   (a) 12, 18 (b) 15, 48 (c) 25, 50, 100 (d) 24, 36, 48

Test Yourself

1. Write down
   (a) the smallest prime number,
   (b) the only even prime number,
   (c) the largest prime number less than 100,
   (d) the prime numbers less than 100 and contain a digit three.

2. Express 48 as the sum of two primes in as many different ways as possible.
   An example is $48 = 5 + 43$.
   (***Hint:*** There are four other ways.)

3. Try this 'crossword' puzzle on multiples, HCF and LCM.



Across
1. LCM of 6 and 12.
2. Multiple of 5, but not of 4.
3. HCF of 48 and 64.
6. LCM of 12 and 40.
8. Within first 5 multiples of 75.
9. Smallest three-digit common multiple of 9 and 12.

Down
1. LCM of 4, 9, 10.
2. HCF of 66 and 88.
4. Multiple of 203.
5. Multiple of 12.
7. HCF of 30 and 45.
10. Within first 3 multiples of 16.

Maths Vitamins

Did you know that all positive even integers greater than 2 can be written as the sum of two primes?

e.g. $4 = 2 + 2$
$6 = 3 + 3$
$8 = 3 + 5$
$10 = 3 + 7 = 5 + 5$

The above statement is known as *Goldbach Conjecture*, first mentioned by the Prussian mathematician Christian Goldbach in 1742 and is one of the oldest unsolved problems in Mathematics. Between March 20 2000 and March 20 2002, *Faber and Faber* offered a \$1 million prize to anyone who can prove *Goldbach Conjecture*, but the prize went unclaimed and the conjecture remains open.

1.3 Integers and Negative Numbers

Maths Vitamins

What is the temperature in the freezer of your refrigerator at home?

Use an appropriate thermometer to check the temperature.

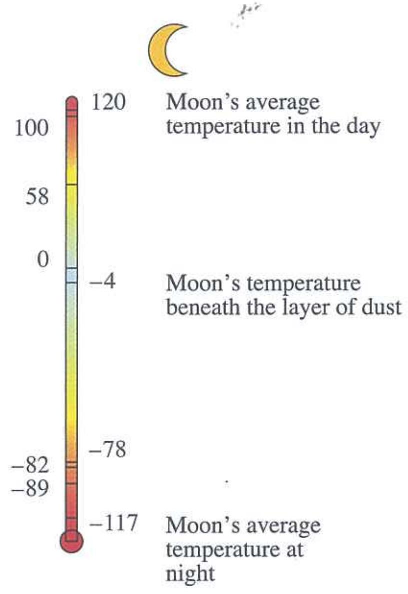
Do you know that the temperature in the chiller section of a supermarket is set at 4 °C?

However, temperatures can go lower than 4 °C. During winter, the temperature can even go below 0 °C in some countries. When it does, we write it as a **negative number**.

For example, when the temperature is 10 °C below 0 °C, we record it as –10 °C.

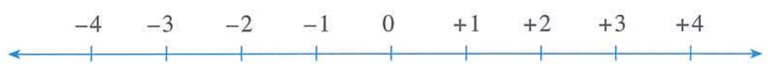
Near the South Pole in Antarctica, an outdoor thermometer once showed an abnormally cold temperature of –87 °C.

The following is another illustration of the use of negative numbers.



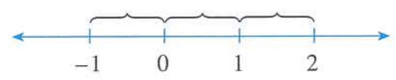
Moon's temperature range in degree Celsius (°C)

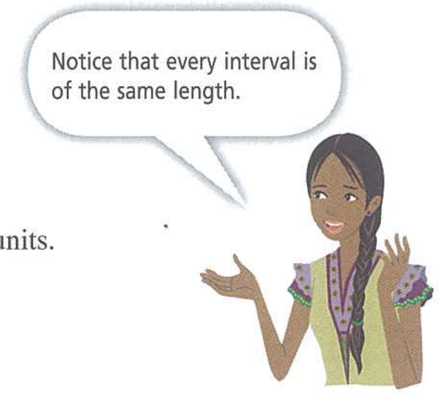
From the above, we observe that numbers can also be negative. These numbers can be represented by points on a line called a **number line** as shown below.



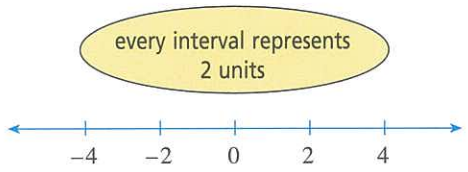
On any number line,

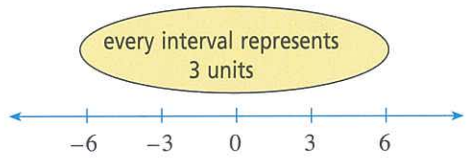
- the interval between any integer and the next has the same length.





- each of these equal intervals can represent any number of units.

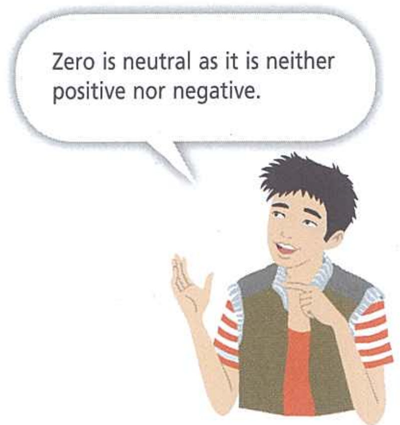




- numbers to the right of zero are positive whole numbers; we attach a '+' sign in front of each of these numbers, i.e. +1, +2, +3, +4, … These numbers are referred to as **positive integers**.

- numbers to the left of zero are negative whole numbers; we attach a '–' sign in front of each of these numbers, i.e. –1, –2, –3, –4, … These numbers are referred to as **negative integers**.

Note that the set of integers consists of positive integers, the number zero and negative integers.



EXAMPLE 6

Mark on the given number line the following.

(a) Positive integers less than 5.

(b) Integers between –2 and 3, excluding the end points.

SOLUTION

(a)

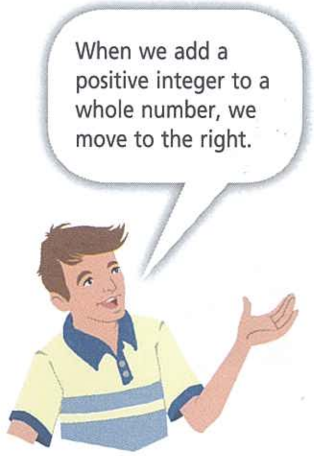
(b)

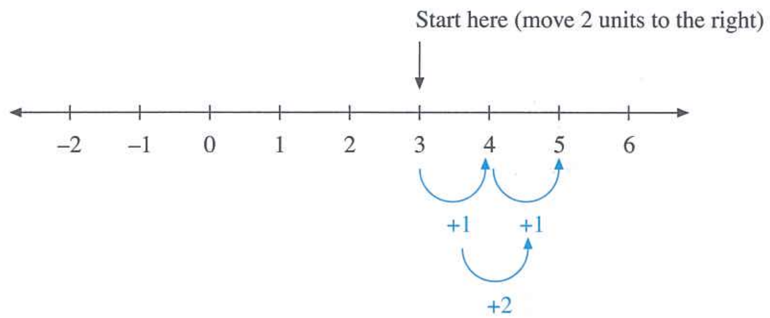
1.4 The Four Operations on Integers

Addition and Subtraction

Addition

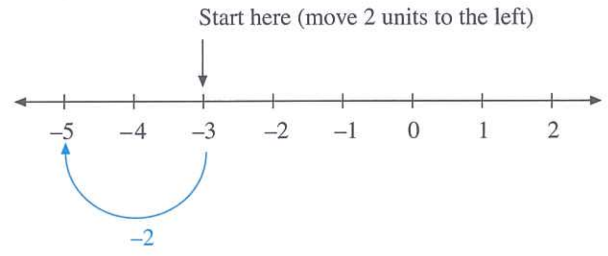
Consider 3 + 2. On the number line, we start from 3 and move 2 units to the right.

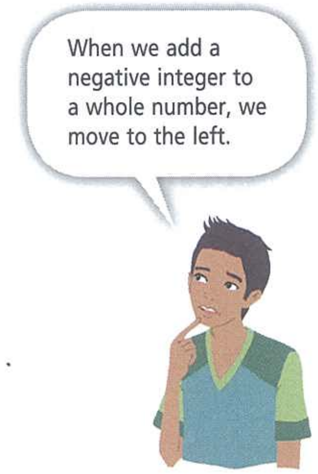




Thus, 3 + 2 = 5.

Consider (–3) + (–2). On the number line, we start from –3 and move 2 units to the left.



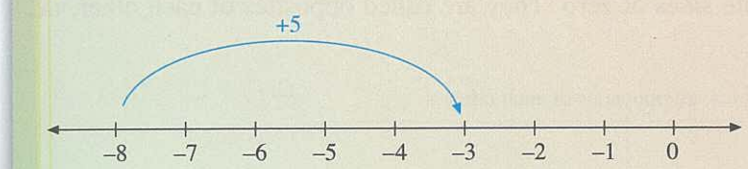


Thus, (–3) + (–2) = –5.

EXAMPLE 7

Show the addition (–8) + 5 using a number line.

SOLUTION

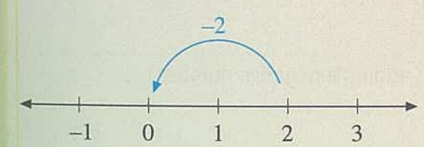


Thus, (–8) + 5 = –3.

EXAMPLE 8

Show the addition 2 + (–2) using a number line.

SOLUTION

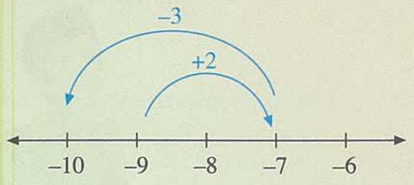


Thus, 2 + (–2) = 0.

EXAMPLE 9

Show the addition (–9) + 2 + (–3) on a number line.

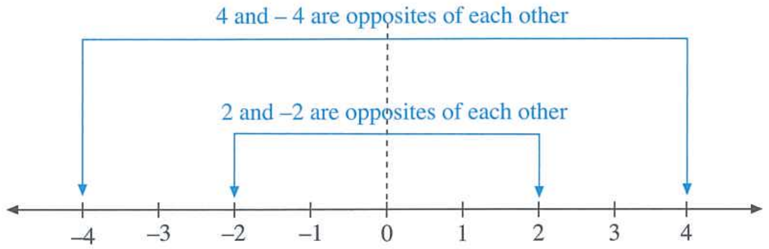
SOLUTION



(–9) + 2 + (–3)
= –7 + (–3)
= –10

Subtraction

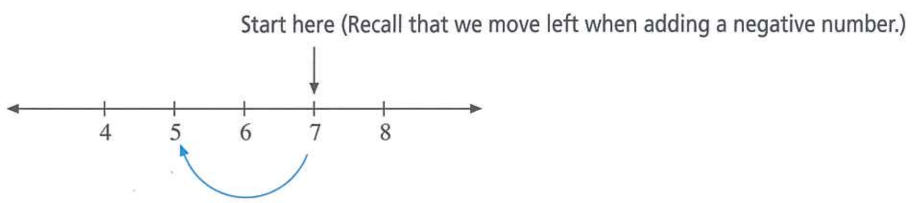
Consider the pairs of numbers 2, –2 and 4, –4. On the number line, the numbers in each pair are at equal distance from zero but lie on opposite sides of zero. They are called opposites of each other, i.e. –2 is the opposite of 2.



When subtracting an integer, we add its opposite.
Take for example, the operation 7 – 2. We can rewrite it as follows.

$$7 - 2 = 7 + \underbrace{(-2)}_{\text{opposite of 2}}$$

We start from 7 on the number line.



Thus, 7 – 2 = 7 + (–2) = 5.

In general, to subtract an integer, we add its opposite,
i.e. $a - b = a + (-b)$

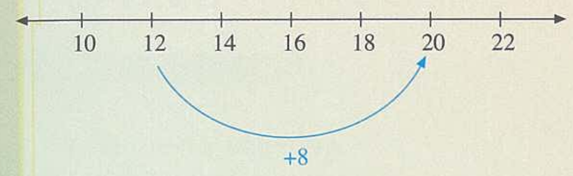
Consider 7 – (–2), the opposite of (–2) is 2. Thus, to subtract (–2) from 7, we add its opposite to 7.
i.e. 7 – (–2) = 7 + 2 = 9

Therefore, when subtracting a negative integer, we have $a - (-b) = a + b$.

EXAMPLE 10

Find 12 – (– 8) using a number line.

SOLUTION

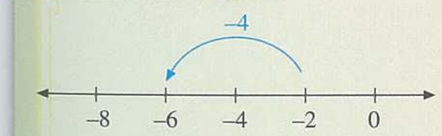


$$12 - (-8) = 12 + \underbrace{(+8)}_{\text{opposite of }-8}$$
$$= 12 + 8$$
$$= 20$$

EXAMPLE 11

Evaluate –2 – 4 using a number line.

SOLUTION



$$-2 - 4 = -2 + \underbrace{(-4)}_{\text{opposite of 4}}$$
$$= -6$$

EXAMPLE 12

Evaluate –3 – (–4).

SOLUTION

$$-3-(-4) = -3 + \overbrace{(4)}^{\text{opposite of } -4}$$
$$= -3 + 4$$
$$= 1$$

EXAMPLE 13

Find the value of each of the following.

(a) –1 – (–2) – (–3) (b) –19 – (–3) + (–8)

SOLUTION

(a)
$$-1 \underbrace{-(-2)} - (-3)$$
$$= -1 \quad +2 \quad -(-3)$$
$$= 1 + 3$$
$$= 4$$

(b)
$$-19 \underbrace{-(-3)} + (-8)$$
$$= \underbrace{-19 + 3} + \underbrace{(-8)}$$
$$= \quad -16 \quad\quad -8$$
$$= -24$$

Alternative solution using a calculator

(a) Press [+/–] [1] [–] [(] [+/–] [2] [)] [–] [(] [+/–] [3] [)] [=]

Final Display: | 4 |

Thus, –1 – (–2) – (–3) = 4

(b) Press [+/–] [1] [9] [–] [(] [+/–] [3] [)] [+] [(] [+/–] [8] [)] [=]

Final Display: | –24 |

Thus, –19 – (–3) + (–8) = –24

Go to Workbook Activity 3, page 5
Go to Workbook Exercise 3, page 14

Practice 1C

Basic

1. If the temperature is originally at 9°C, what will the final temperature be if the temperature changes by the following degrees?
   (a) +3°C (b) –12°C (c) –8°C (d) +5°C

2. With the aid of a number line, evaluate the following.
   (a) 8 – 4 (b) 2 – (–3) (c) 5 – (–3) (d) –7 – (–5) (e) –4 – (–8)

Advanced

3. Evaluate the following.
   (a) –8 – (–4) + 5 (b) 5 + (–10) – 6 (c) – 6 – 8 – 12 (d) –23 + (–17) + 7
   (e) –20 + 16 – 7 (f) –11 + (–8) + 14

Applications

4. Mr Maidin has two freezers. One keeps frozen goods at a temperature of –29°C, while the other keeps goods at a temperature of –16°C.
   He transferred a packet of frozen food from one freezer to the other.
   (a) What is the temperature difference between the two freezers?
   (b) If the temperature of the packet rises after the transfer, from which freezer is the packet taken?

Let's Explore

(a) An operation is **commutative** if the result of the operation does not depend on the order that the operation is carried out on the individual integers.

The addition of positive integers is commutative. (Recall that $2 + 5 = 5 + 2$)

Complete the table below and deduce whether the addition of negative integers is also commutative.

Column A	Column B	Is Column A = Column B?
$8 + (-9) =$	$(-9) + 8 =$	
$-7 + (-2) =$	$(-2) + (-7) =$	
$-9 + (-12) =$	$(-12) + (-9) =$	
$-6 + 5 =$	$5 + (-6) =$	

(b) An operation is **associative** if the result of the operation does not depend on how the individual integers are grouped.

The addition of positive integers is associative. (Recall that $3 + (2 + 1) = (3 + 2) + 1$)

Complete the table below and deduce whether the addition of negative integers is also associative.

Column A	Column B	Is Column A = Column B?
$8 + (-9) =$	$(-9) + 8 =$	
$[9 + (-11)] + (-12) =$	$9 + [(-11) + (-12)] =$	
$-12 + [4 + (-7)] =$	$[(-12) + 4] + (-7) =$	
$-2 + [-9 + (-8)] =$	$[-2 + (-9)] + (-8) =$	

(c) Determine whether the subtraction of negative integers is commutative as well as associative.

Multiplication and Division

Multiplication

Study the following patterns and fill in the blanks provided.

$3 \times 3 = 9$	$(-3) \times 3 = -9$
$3 \times 2 = 6$	$(-3) \times 2 = -6$
$3 \times 1 = 3$	$(-3) \times 1 =$ ______
$3 \times 0 = 0$	$(-3) \times 0 =$ ______
$3 \times (-1) =$ ______	$(-3) \times (-1) =$ ______
$3 \times (-2) =$ ______	$(-3) \times (-2) =$ ______

(a) What do you observe about the sign of the product of a positive number and a negative number?

(b) What do you observe about the sign of the product of a negative number and a negative number?

(c) Fill in the blanks based on the above observations.

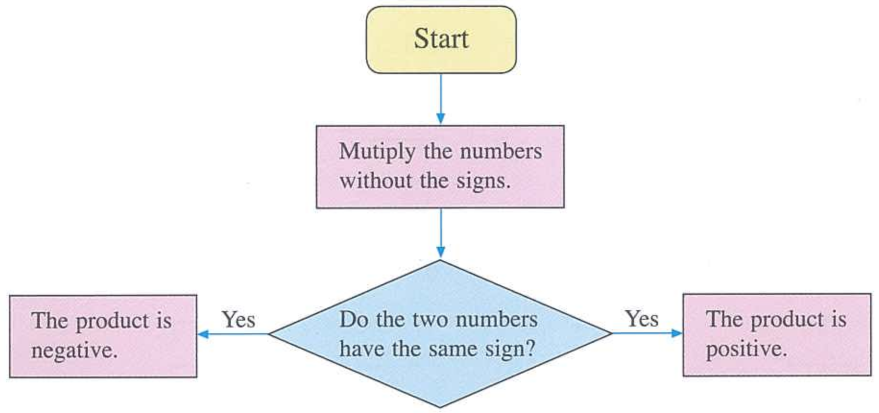
When a and b are positive integers,

$(+a) \times (-b) =$ ______ $\qquad (-a) \times (-b) =$ ______

Thus, the rules for the multiplication of integers are as follows.

- When multiplying two numbers of different signs, the answer is negative.
  e.g. $(+1)(-1) = -1$; $\quad (-1)(+1) = (-1)$
- When multiplying two numbers of the same sign, the answer is positive.
  e.g. $(+1)(+1) = +1$; $\quad (-1)(-1) = +1$

We can use the following flow chart for the multiplication of two non-zero integers.



In general, for any two positive integers a and b,

$$(-a) \times (-b) = ab$$
$$a \times (-b) = -ab$$
$$(-a) \times b = -ab$$

EXAMPLE 14

Find the values of

(a) $(-4) \times (-5)$ (b) $9 \times (-3)$ (c) $2 \times (-1) \times (-3)$

SOLUTION

(a) $(-4) \times (-5) = 20$

(b) $9 \times (-3) = -27$

(c) $2 \times (-1) \times (-3) = (-2) \times (-3) = 6$

Division

Study the following patterns and fill in the blanks provided.

Multiplication	**Division**
$3 \times (-1) = -3$	$(-3) \div 3 = -1$
$3 \times (-2) = -6$	$(-6) \div 3 = -2$
$3 \times (-3) = -9$	$(-9) \div 3 =$ ______
$3 \times (-4) =$ ______	______ $\div 3 =$ ______
$(-3) \times 4 = -12$	$(-12) \div (-3) = 4$
$(-3) \times 3 = -9$	$(-9) \div (-3) =$ ______
$(-3) \times 2 =$ ______	______ $\div (-3) =$ ______
$(-3) \times 1 =$ ______	______ $\div (-3) =$ ______
$(-3) \times (-1) = 3$	$3 \div (-3) = -1$
$(-3) \times (-2) = 6$	$6 \div (-3) =$ ______
$(-3) \times (-3) =$ ______	______ $\div (-3) =$ ______
$(-3) \times (-4) =$ ______	______ $\div (-3) =$ ______

(a) What do you observe about the sign of the result when a negative number is divided by a positive number?

(b) What do you observe about the sign of the result when a negative number is divided by a negative number?

(c) What do you observe about the sign of the result when a positive number is divided by a negative number?

(d) Fill in the blanks based on the above observations.

When a and b are positive integers,

$$(-a) \div b = -\frac{a}{b}$$

$$(-a) \div (-b) = \text{\_\_\_\_\_\_}$$

$$a \div (-b) = \text{\_\_\_\_\_\_}$$

Thus, the rules for division of integers are as follows.

- When dividing two numbers of different signs, the answer is negative.

  e.g. $(-1) \div (+1) = -1$ and $(+1) \div (-1) = -1$

- When dividing two numbers of the same sign, the answer is positive.

  e.g. $(+1) \div (+1) = (+1)$ and $(-1) \div (-1) = (+1)$

In general, for any positive integers a and b,

$$(-a) \div b = -\frac{a}{b}$$

$$(-a) \div (-b) = \frac{a}{b}$$

$$a \div (-b) = -\frac{a}{b}$$

EXAMPLE 15

Find the values of

(a) $16 \div 2$ (b) $(-25) \div (-5)$ (c) $(-81) \div 3$ (d) $96 \div (-4)$

SOLUTION

(a) $16 \div 2 = 8$

(b) $(-25) \div (-5) = 5$

(c) $(-81) \div 3 = -27$

(d) $96 \div (-4) = -24$

EXAMPLE 16

Evaluate the following.

(a) (–15) ÷ (–3) + (–2) × 8

(b) [(–13) – (–5)] × 12 ÷ (–84 + 52)

SOLUTION

(a) Using a calculator,

Press [( ] [+/–] [1] [5] [ )] [÷] [( ] [+/–] [3] [ )] [+] [( ] [+/–] [2] [ )] [×] [8] [=]

Final Display: –11

Thus, –15 ÷ (–3) + (–2) × 8 = –11.

(b) Using a calculator,

Press [( ] [( ] [+/–] [1] [3] [ )] [–] [( ] [+/–] [5] [ )] [ )] [×] [1] [2] [÷] [( ] [+/–] [8] [4] [+] [5] [2] [ )] [=]

Final Display: 3

Thus, [(–13) – (–5)] × 12 ÷ (–84 + 52) = 3.

It is important to use the bracket keys [( ] [ )] to ensure that the correct order of operation is carried out.

Go to Workbook Exercise 4, page 16

Practice 1D

Basic

1. Find the products of the following.
   (a) 3 × (–10) (b) –3 × (–10) (c) –5 × 8
   (d) –4 × 0 (e) 10 × (–7) (f) 5 × (–1) × (–4)
   (g) (–13) × (–2) × 4 (h) (–5) × (–3) × (–2) (i) 8 × (–9) × 0

2. Evaluate the following.
   (a) –36 ÷ 4 (b) –100 ÷ 5 (c) 72 ÷ (–8)
   (d) 81 ÷ (–3) (e) 0 ÷ (–2) (f) –144 ÷ (–12)
   (g) –28 ÷ (–4) (h) 128 ÷ (–8)

Advanced

3. Complete the following.
   (a) 9 × ☐ = –45 (b) ☐ × (–4) = 128 (c) –5 × ☐ = 0
   (d) ☐ × 9 = –99 (e) ☐ × (–3) × (–2) = 36 (f) –9 × (–1) × ☐ = 54

4. Complete the following.
   (a) ☐ ÷ (–5) = 3 (b) ☐ ÷ (–4) = –5 (c) ☐ ÷ 8 = –9
   (d) 108 ÷ ☐ = –27 (e) –121 ÷ ☐ = –11 (f) –240 ÷ ☐ = 120

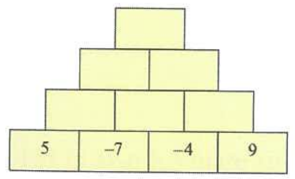
5. Evaluate the following with the help of a calculator.
   (a) [(–91) + 52] × (–12)
   (b) 296 – {[281 + (–125)] × 11}
   (c) [–72 – (–12)] × 3 + [275 ÷ (–25)]
   (d) {2[540 – 260 – (–38)] – 153} – 48 × [(–16) + 27]

Test Yourself

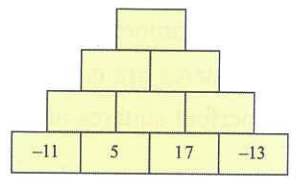
1. State whether each of the following statements is true or false.
   (a) On a horizontal number line, larger numbers are usually graphed to the right of the smaller numbers.
   (b) The sum of two negative integers is negative.
   (c) The product of a positive integer and a negative integer is positive.
   (d) The product of two negative integers is positive.
   (e) The smallest number of the list –8, 0, 8, –10 is 0.

2. Place a number in each empty box so that the number in the box will be the sum of the pair of numbers beneath the box.

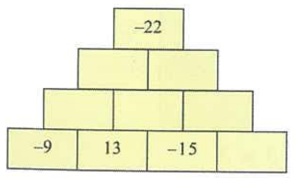
(a)



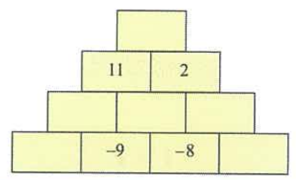
(b)



(c)



(d)



CHAPTER 1 : NUMBERS

Let's Write

Have you ever wondered when negative numbers were first used and how they were represented?

Historically, the Chinese had some knowledge of negative numbers as early as 200 B.C. The Hindu Brahmagupta stated the rules of operations with positive and negative numbers in the seventh century.

Research on the origin of negative numbers and write a journal entry of what you learnt.

Suggestion: You may want to visit some websites featuring the history of Mathematics to find out more information about negative numbers and consider the following questions during your search.

- When were negative numbers first used?
- How were negative numbers first represented?
- What was man's view of negative numbers prior to the 1600's?
- What were the practical problems encountered in using negative numbers?

Go to Workbook Activity 4, page 6

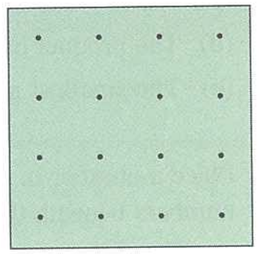
1.5 Squares and Square Roots

Squares

Recall that 16 can be written in index form as 4^2, which means 4×4, and is read as the **square** of 4. In other words, we say that 16 is the square of 4.

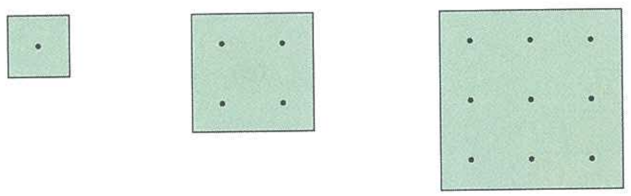
What is the square of 5?

The number 16 can be represented by a square shown on the right:



4×4

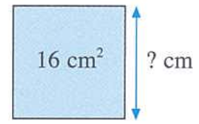
Can you think of other numbers that can also form squares?
Numbers that form squares are called **perfect squares**.
More examples of perfect squares are shown below.



1×1 2×2 3×3

Square roots

The diagram below shows a square with area 16 cm^2.



What is the length of one side of the square?

i.e. ____ × ____ = 16

Since $4 \times 4 = 4^2 = 16$, the length of one side of the square is 4 cm.

Thus, we say, 4 is the positive **square root** of 16 and we write $\sqrt{16} = \sqrt{4 \times 4} = 4$.

In this case, the square root of 16 cannot be a negative value since we cannot have a negative length.

In general, '$\sqrt{16}$' is the symbol to mean positive square root.

In addition, (-4) is also a square root of 16 as $(-4) \times (-4) = (-4)^2 = 16$.

Therefore, ± 4 are the two square roots of 16.

To find the square root of a number, we can also use the prime factorisation method.

For example,

$$\begin{aligned} 225 &= 3 \times 3 \times 5 \times 5 \\ &= (3 \times 5) \times (3 \times 5) \\ &= (3 \times 5)^2 \\ &= 15^2 \end{aligned}$$

However (-15) is also a square root of 225 as $(-15) \times (-15) = 225$.

Therefore, the two square roots of 225 are ± 15.

Take note that 16 is the square of −4. Thus −4 is also a square root of 16. Hence 16 has a positive square root and a negative square root.

Example 17

Find the two square roots of 196 using prime factorisation.

SOLUTION

$$\pm\sqrt{196} = \pm\sqrt{2 \times 2 \times 7 \times 7}$$
$$= \pm\sqrt{(2 \times 7) \times (2 \times 7)}$$
$$= \pm(2 \times 7)$$
$$= \pm 14$$

Example 18

Find the value of each of the following.

(a) $\sqrt{6084}$ (b) $\sqrt{13\,456}$

SOLUTION

Press [√] [6] [0] [8] [4] [=]

Final Display: 78

Thus, $\sqrt{6084} = 78$.

(b) Using a calculator,

Press [√] [1] [3] [4] [5] [6] [=]

Final Display: 116

Thus, $\sqrt{13\,456} = 116$.

Go to Workbook Activity 5, page 7
Go to Workbook Exercise 5, page 19

Practice 1E

Basic

1. Write down the square of each of the following integers.
   (a) 8 (b) 12 (c) 14 (d) –7 (e) –11

2. Using prime factorisation, find the square roots of each of the following. (Include negative roots)
   (a) 256 (b) 324 (c) 400 (d) 441 (e) 625

Advanced

3. Using prime factorisation, state which of the following are squares of whole numbers.
   (a) 576 (b) 1152 (c) 729 (d) 1296 (e) 1734

4. Find the value(s) of each of the following.
   (a) $\sqrt{1521}$ (b) $\pm\sqrt{4225}$ (c) the square roots of 17 424
   (d) the square roots of 80 656

1.6 Cubes and Cube Roots

In the previous section, we learnt that $4 \times 4 = 4^2 = 16$ and $\sqrt{16} = 4$.

Similarly, $4 \times 4 \times 4 = 4^3 = 64$

We say that:
(i) the **cube** of 4 is 64, while
(ii) 4 is the **cube root** of 64.

> **Think Maths**
> What is the cube of –4? Do you get 64? If not, what is your answer?

We write the cube root of 64 as $\sqrt[3]{64} = \sqrt[3]{4 \times 4 \times 4} = 4$.

We can again use the prime factorisation method to find the cube root of a number.

For example,

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$
$$= (2 \times 3) \times (2 \times 3) \times (2 \times 3)$$
$$= (2 \times 3)^3$$
$$= 6^3$$

Thus, $\sqrt[3]{216} = 6$.

EXAMPLE 19

Find the value of $\sqrt[3]{512}$ by using prime factorisation.

SOLUTION

$$\begin{aligned}\sqrt[3]{512} &= \sqrt[3]{(2\times2\times2)\times(2\times2\times2)\times(2\times2\times2)}\\ &= 2\times2\times2\\ &= 8\end{aligned}$$

EXAMPLE 20

Find the cube root of –1728.

SOLUTION

$$\begin{aligned}\text{The cube root of } -1728 &= \sqrt[3]{-1728}\\ &= \sqrt[3]{(-1)\times(1728)}\\ &= \sqrt[3]{[(-1)\times(-1)\times(-1)]\times(1728)}\\ &= (-1)\times\sqrt[3]{1728}\\ &= -\sqrt[3]{2\times2\times2\times2\times2\times2\times3\times3\times3}\\ &= -\sqrt[3]{(2\times2\times3)\times(2\times2\times3)\times(2\times2\times3)}\\ &= -(2\times2\times3)\\ &= -12\end{aligned}$$

EXAMPLE 21

Find the cube root of

(a) 4913 (b) –21 952

SOLUTION

(a) Using a calculator,

Press [$\sqrt[3]{\ }$] [4] [9] [1] [3] [=]

Final Display: 17

Thus, $\sqrt[3]{4913} = 17$.

(b) Using a calculator,

Press [$\sqrt[3]{\ }$] [+/–] [2] [1] [9] [5] [2] [=]

Final Display: –28

Thus, $\sqrt[3]{-21\,952} = -28$

Go to Workbook Exercise 6, page 20

Practice 1F

Basic

1. Find the cube of each of the following.
   (a) 9 (b) 11 (c) 13 (d) –5 (e) –10

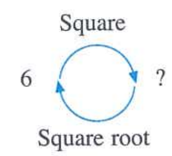
2. Using prime factorisation, state which of the following are cubes of whole numbers.
   (a) –64 (b) 343 (c) 1728 (d) –729 (e) 1975

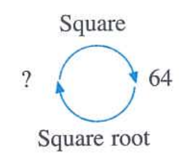
3. Using prime factorisation, find the cube root of each of the following.
   (a) 64 (b) 125 (c) –512 (d) 2197 (e) –1331

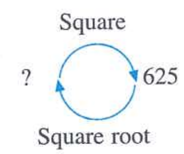
4. Find the cube root of each of the following.
   (a) 2744 (b) –9261 (c) 42 875 (d) –74 088

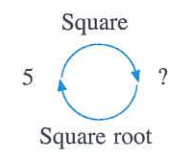
Test Yourself

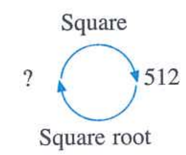
1. Find the missing numbers for the following.

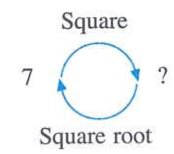
(a)


(b)


(c)


(d)


(e)


(f)


2. Claudine did an exercise on square roots and cube roots as shown below. Check her calculations by squaring or cubing the answers. Put a tick (✓) for the correct answer and a cross (✗) for the wrong answer.

Do not use the $\sqrt[3]{\ }$ key of your calculator. Show your working. The answer for (a) has been completed as an example.

Example	$\sqrt[3]{216} = 6$
Working	$6^3 = 6 \times 6 \times 6 = 216$
Answer	$\sqrt[3]{216} = 6$ ✓

Claudine

(a)	$\sqrt[3]{216} = 6$
(b)	$\sqrt[3]{64} = 8$
(c)	$\sqrt{169} = 13$
(d)	$\sqrt{289} = 17$
(e)	$\sqrt[3]{8000} = 40$
(f)	$\sqrt[3]{1331} = 11$

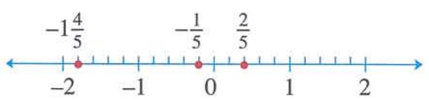
1.7 Real Numbers and Rational Numbers

Real Numbers

Real numbers are the numbers that can be represented on a number line, that is, a real number has only one fixed point on the number line.

Go to Workbook Activity 6, page 8

From the activity, one can see that there are many other numbers in between any two integers. Since a number may also be negative, the number line can be extended to the left as shown below.



The length between two integers (e.g. 1 and 2) on the number line shown is divided into 5 equal parts.

The numbers $\frac{2}{5}$, $-\frac{1}{5}$ and $-1\frac{4}{5}$ have been marked on the number line. Can you also mark the numbers $-\frac{3}{5}$ and $-1\frac{1}{5}$ on this number line?

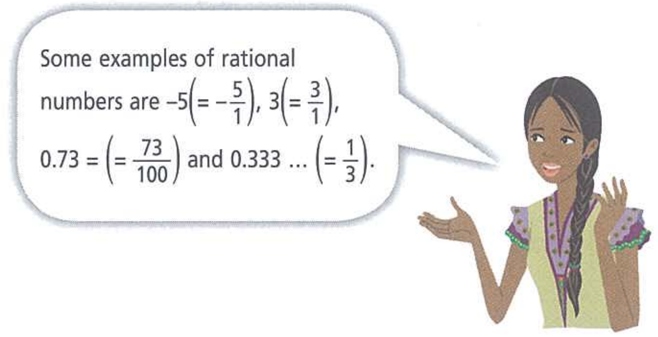
In fact, an infinite quantity of such numbers can be marked on this number line. For example, $-10\frac{1}{2}$, $-8\frac{3}{4}$, -0.4, $35\frac{1}{5}$, $36\frac{1}{4}$, 37.8, 38.9, etc.

Rational Numbers

The numerical values mentioned on page 30 are rational numbers.

So, what is a rational number?

A **rational** number is a number that can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.



Rational numbers include decimals that can either be terminating such as 0.73 or recurring such as 0.333… since they can be written as $\frac{73}{100}$ and $\frac{1}{3}$ respectively.

For example,

$-2\frac{1}{5} = -\frac{11}{5} = -2.2$ (terminating decimal)

$$\begin{array}{r} 2.2 \\ 5\overline{)\,1\,1} \\ -1\,0 \\ \hline 1\,0 \\ 1\,0 \\ \hline 0 \end{array}$$

$\frac{1}{4} = 0.25$ (terminating decimal)

$$\begin{array}{r} 0.2\,5 \\ 4\overline{)\,1\,0} \\ -\ \ 8 \\ \hline 2\,0 \\ 2\,0 \\ \hline 0 \end{array}$$

$\frac{2}{3} = 0.666\ldots = 0.\dot{6}$ (recurring decimal)

$$\begin{array}{r} 0.6\,6\,6\ldots \\ 3\overline{)\,2\,0} \\ -1\,8 \\ \hline 2\,0 \\ 1\,8 \\ \hline 2 \end{array}$$

$\frac{3}{7} = 0.428\ 571\ 428\ 571\ 4\ldots$

$= \dot{4}28\ 57\dot{1}$ (recurring decimal)

$$
\begin{array}{r}
0.4\,2\,8\,5\,7\,1\,4 \\
7\,\big)\ \ 3\,0 \\
-\ 2\,8 \\
\hline
2\,0 \\
1\,4 \\
\hline
6\,0 \\
5\,6 \\
\hline
4\,0 \\
3\,5 \\
\hline
5\,0 \\
4\,9 \\
\hline
1\,0 \\
7 \\
\hline
3\,0 \\
2\,8 \\
\hline
2
\end{array}
$$

Since the pattern from digits 4 to 1 is recurring, we place dots above the first and last digits of the repeating pattern.

Not all numbers on a number line can be written in the form $\frac{a}{b}$.
e.g. $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, etc.

To summarise, real numbers consist of both rational and irrational numbers.

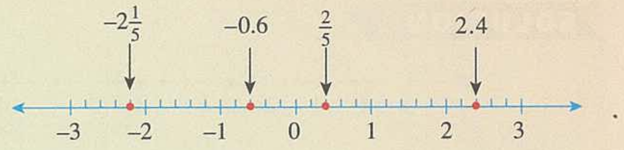
- Real numbers
  - Rational numbers
    - Fractions
    - Integers
      - Negative integers
      - Whole numbers
        - Zero
        - Positive integers
  - Irrational numbers

EXAMPLE 22

Mark each of the following points on a number line. Then arrange them in ascending order.

(a) $-2\frac{1}{5}$ (b) 2.4 (c) $\frac{2}{5}$ (d) -0.6

SOLUTION



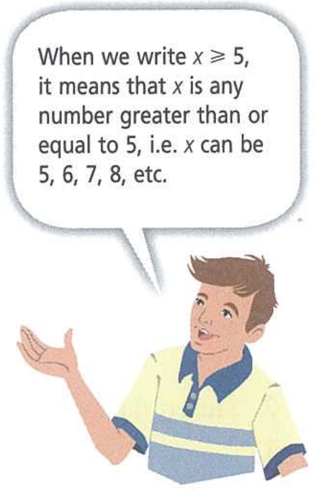
The number line shows the order in value of the numbers in ascending order (from the smallest to the greatest). The numbers arranged in ascending order are as follows.

$-2\frac{1}{5}$, -0.6, $\frac{2}{5}$, 2.4

Use of the Symbols '>', '<', '≥', '≤'

Inequality symbols can be used to show the order or compare the values of real numbers.

The symbol '<' means 'is less than'.
The symbol '>' means 'is more than'.
The symbol '≤' means 'is less than or equal to'.
The symbol '≥' means 'is more than or equal to'.



EXAMPLE 23

Compare each of the following using '<' or '>'.

(a) 18 ☐ 20 (b) -16 ☐ -10 (c) 0 ☐ -7 (d) 8 ☐ 0

(e) $-3\frac{1}{2}$ ☐ 5 (f) $-14\frac{1}{4}$ ☐ $-17\frac{1}{4}$ (g) $\frac{2}{3}$ ☐ 0.6

SOLUTION

(a) $18 < 20$ (b) $-16 < -10$ (c) $0 > -7$ (d) $8 > 0$

(e) $-3\frac{1}{2} < 5$ (f) $-14\frac{1}{4} > -17\frac{1}{4}$ (g) $\frac{2}{3} > 0.6$

Let's communicate

We have seen negative numbers being used in the measurements of temperature. There are many more other instances where negative numbers are used. Think of as many examples as you can, of such instances and share with your classmates.

EXAMPLE 24

Arrange the following real numbers in ascending order (i.e. from the smallest to the greatest).

$11.5, -2\frac{1}{3}, 3, -5\frac{3}{4}, 4, 0, -8, 7.6$

SOLUTION

The real numbers arranged in ascending order are as follows.

$-8, -5\frac{3}{4}, -2\frac{1}{3}, 0, 3, 4, 7.6, 11.5$

Go to Workbook Exercise 7, page 21

Practice 1G

Basic

1. Mark the following numbers on a number line.

   (a) $-2.5, \frac{1}{2}, -1\frac{1}{2}, 1.\dot{4}\dot{1}, 2.5, -3.1$ (b) $\frac{1}{2}, -\frac{1}{2}, 1.5, -1.7, -1\frac{3}{4}, 1.75$

2. Insert the symbol '>' or '<' in each of the following to make the sentences true.

   (a) $-\frac{1}{2}$ ☐ $\frac{1}{4}$ (b) $-2\frac{1}{4}$ ☐ $-\frac{1}{2}$ (c) -1.7 ☐ 2

   (d) 1.5 ☐ $\frac{3}{4}$ (e) 0 ☐ $-\frac{1}{3}$ (f) $-1\frac{2}{5}$ ☐ -1

   (g) 0.08 ☐ 0.1 (h) -5 ☐ $-(-5)$ (i) -0.3 ☐ -0.5

EXAMPLE 25

Evaluate the following.

(a) $\frac{5}{9} + \frac{3}{4} + \frac{1}{6}$ (b) $\frac{27}{30} - \frac{2}{15} - \frac{3}{10}$

SOLUTION

(a)
$$\begin{aligned}\frac{5}{9} + \frac{3}{4} + \frac{1}{6} &= \frac{5 \times 4}{9 \times 4} + \frac{3 \times 9}{4 \times 9} + \frac{1 \times 6}{6 \times 6}\\ &= \frac{5 \times 4 + 3 \times 9 + 1 \times 6}{36}\\ &= \frac{53}{36}\\ &= 1\frac{17}{36}\end{aligned}$$

Alternatively, using a calculator,

Press [5] [a b/c] [9] [+] [3] [a b/c] [4] [+] [1] [a b/c] [6] [=]

Final display: 1 ⌟ 17 ⌟ 36

Thus, $\frac{5}{9} + \frac{3}{4} + \frac{1}{6} = 1\frac{17}{36}$.

(b)
$$\begin{aligned}\frac{27}{30} - \frac{2}{15} - \frac{3}{10} &= \frac{27}{30} - \frac{2 \times 2}{15 \times 2} - \frac{3 \times 3}{10 \times 3}\\ &= \frac{27 - 4 - 9}{30}\\ &= \frac{14}{30}\\ &= \frac{7}{15}\end{aligned}$$

Alternatively, using a calculator,

Press [2] [7] [a b/c] [3] [0] [−] [2] [a b/c] [1] [5] [−] [3] [a b/c] [1] [0] [=]

Final display: 7 ⌟ 15

Thus, $\frac{27}{30} - \frac{2}{15} - \frac{3}{10} = \frac{7}{15}$.

EXAMPLE 26

Evaluate

(a) $5\frac{3}{5} + 2\frac{13}{15}$ (b) $4\frac{4}{7} + 1\frac{2}{7} + 3\frac{13}{42}$

SOLUTION

(a) $$5\frac{3}{5} + 2\frac{13}{15} = \frac{28}{5} + \frac{43}{15} = \frac{28 \times 3 + 43}{15} = \frac{127}{15} = 8\frac{7}{15}$$

Alternatively, using a calculator,

Press 5 a b/c 3 a b/c 5 + 2 a b/c 1 3 a b/c 1 5 =

Final display: 8 ⌟ 7 ⌟ 15

Thus, $5\frac{3}{5} + 2\frac{13}{15} = 8\frac{7}{15}$.

(b) $$4\frac{4}{7} + 1\frac{2}{7} + 3\frac{13}{42} = \frac{32}{7} + \frac{9}{7} + \frac{139}{42} = \frac{32 \times 6 + 9 \times 6 + 139}{42} = \frac{385}{42} = 9\frac{1}{6}$$

Alternatively, using a calculator,

Press 4 a b/c 4 a b/c 7 + 1 a b/c 2 a b/c 7 + 3 a b/c 1 3 a b/c 4 2 =

Final display: 9 ⌟ 1 ⌟ 6

Thus, $4\frac{4}{7} + 1\frac{2}{7} + 3\frac{13}{42} = 9\frac{1}{6}$.

EXAMPLE 27

Evaluate

(a) $4\frac{1}{6} - 2\frac{2}{3} - 1\frac{1}{24}$ (b) $10\frac{5}{6} + 6\frac{8}{9} - 9\frac{11}{18}$

SOLUTION

(a) $$4\frac{1}{6} - 2\frac{2}{3} - 1\frac{1}{24} = \frac{25}{6} - \frac{8}{3} - \frac{25}{24} = \frac{25 \times 4 - 8 \times 8 - 25}{24} = \frac{11}{24}$$

Alternatively, using a calculator,

Press 4 a b/c 1 a b/c 6 − 2 a b/c 2 a b/c 3 − 1 a b/c 1 a b/c 2 4 =

Final display: 11 ⌟ 24

Thus, $4\frac{1}{6} - 2\frac{2}{3} - 1\frac{1}{24} = \frac{11}{24}$.

(b) $$10\frac{5}{6} + 6\frac{8}{9} - 9\frac{11}{18} = \frac{65}{6} + \frac{62}{9} - \frac{173}{18} = \frac{65 \times 3 + 62 \times 2 - 173}{18} = \frac{146}{18} = 8\frac{1}{9}$$

Alternatively, using a calculator,

Press 1 0 a b/c 5 a b/c 6 + 6 a b/c 8 a b/c 9 − 9 a b/c 1 1 a b/c 1 8 =

Final display: 8 ⌟ 1 ⌟ 9

Thus, $10\frac{5}{6} + 6\frac{8}{9} - 9\frac{11}{18} = 8\frac{1}{9}$.

EXAMPLE 28

Simplify (a) $2\frac{3}{5}\times 5\frac{1}{3}\times\frac{3}{4}$ (b) $3\frac{3}{5}\times 1\frac{4}{9}-3\frac{1}{4}\div 1\frac{5}{8}$

SOLUTION

(a) $$2\frac{3}{5}\times 5\frac{1}{3}\times\frac{3}{4}=\frac{13}{5}\times\frac{\cancel{16}^{4}}{{}_{1}\cancel{3}}\times\frac{\cancel{3}^{1}}{\cancel{4}_{1}}$$
$$=\frac{13\times 4\times 1}{5\times 1\times 1}$$
$$=\frac{52}{5}$$
$$=10\frac{2}{5}$$

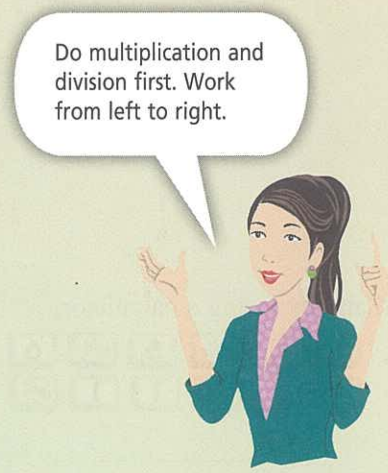
Alternatively, using a calculator,

Press [2] [a b/c] [3] [a b/c] [5] [×] [5] [a b/c] [1] [a b/c] [3]
[×] [3] [a b/c] [4] [=]

Final display: 10 ⌟ 2 ⌟ 5

Thus, $2\frac{3}{5}\times 5\frac{1}{3}\times\frac{3}{4}=10\frac{2}{5}$.

(b) $$3\frac{3}{5}\times 1\frac{4}{9}-3\frac{1}{4}\div 1\frac{5}{8}=\frac{{}^{2}\cancel{18}}{5}\times\frac{13}{\cancel{9}_{1}}-\frac{13}{4}\div\frac{13}{8}$$
$$=\frac{26}{5}-\frac{{}^{1}\cancel{13}}{{}_{1}\cancel{4}}\times\frac{\cancel{8}^{2}}{\cancel{13}_{1}}$$
$$=\frac{26}{5}-2$$
$$=\frac{26-10}{5}$$
$$=\frac{16}{5}$$
$$=3\frac{1}{5}$$



Alternatively, using a calculator,

Press [3] [a b/c] [3] [a b/c] [5] [×] [1] [a b/c] [4] [a b/c] [9] [−]
[3] [a b/c] [1] [a b/c] [4] [÷] [1] [a b/c] [5] [a b/c] [8] [=]

Final display: 3 ⌟ 1 ⌟ 5

Thus, $3\frac{3}{5}\times 1\frac{4}{9}-3\frac{1}{4}\div 1\frac{5}{8}=3\frac{1}{5}$.

EXAMPLE 29

Evaluate the following expressions using a calculator, giving your answers correct to 3 decimal places where necessary.

(a) $\frac{94}{56\times 62}$ (b) $\frac{18.3+34.5}{12.48}$ (c) $2\times\left(3\frac{3}{4}-1\frac{3}{5}\right)$

SOLUTION

(a) Press [9] [4] [÷] [(] [5] [6] [×] [6] [2] [)] [=]

Final display: 0.027073732

Thus, $\frac{94}{56\times 62}=0.027$ (to 3 dec. pl.).

Brackets have to be used to ensure the correct order of operation is carried out. If not, the calculator will divide 94 by 56 before multiplying 62.

(b) Press [(] [1] [8] [·] [3] [+] [3] [4] [·] [5] [)] [÷] [1]
[2] [·] [4] [8] [=]

Final display: 4.230769231

Thus, $\frac{18.3+34.5}{12.48}=4.231$ (to 3 dec. pl.).

In this case, brackets are used to ensure the addition operation is carried out before division.

(c) Press [2] [×] [(] [3] [a b/c] [3] [a b/c] [4] [−] [1] [a b/c] [3]
[a b/c] [5] [)] [=]

Final display: 4 ⌟ 3 ⌟ 10

Thus, $2\times\left(3\frac{3}{4}-1\frac{3}{5}\right)=4\frac{3}{10}$.

EXAMPLE 30

Evaluate the following using a calculator.

(a) $-3.5 \times (16.9 \div 1.3)$ (b) $3\frac{1}{5} \times \left(-1\frac{1}{3}\right) \times \left(-2\frac{1}{2}\right)$

SOLUTION

(a) Press [+/–] [3] [•] [5] [×] [(] [1] [6] [•] [9] [÷] [1] [•] [3] [)] [=]

Final display: $\boxed{-45.5}$

Thus, $-3.5 \times (16.9 \div 1.3) = -45.5$.

(b) Press [3] [a b/c] [1] [a b/c] [5] [×] [(] [+/–] [1] [a b/c] [1] [a b/c] [3] [)] [×] [(] [+/–] [2] [a b/c] [1] [a b/c] [2] [)] [=]

Final display: $\boxed{10 \lrcorner 2 \lrcorner 3}$

Thus, $3\frac{1}{5} \times \left(-1\frac{1}{3}\right) \times \left(-2\frac{1}{2}\right) = 10\frac{2}{3}$.

EXAMPLE 31

Evaluate the following, giving your answers correct to 1 decimal place.

(a) $\sqrt{252} - (-3.6)^3$ (b) $\sqrt{79} + (-29)^2$

SOLUTION

(a) Press [√] [2] [5] [2] [–] [(] [+/–] [3] [•] [6] [)] [x^y] [3] [=]

Final display: $\boxed{62.53050787}$

Thus, $\sqrt{252} - (-3.6)^3 = 62.5$ (to 1 dec. pl.).

(b) Press [√] [7] [9] [+] [(] [+/–] [2] [9] [)] [x^2] [=]

Final display: $\boxed{849.8881944}$

Thus, $\sqrt{79} + (-29)^2 = 849.9$ (to 1 dec. pl.).

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Go to Workbook Exercise 8, page 22
Go to Workbook Project B, page 26

Practice 1H

Basic

1. Evaluate the following.

   (a) $5\frac{3}{4} \times \left(1\frac{1}{2} \div 1\frac{1}{3}\right)$ (b) $6\frac{3}{4} - 2\frac{1}{2} \times 1\frac{4}{5}$ (c) $\left(\frac{1}{2} + 1\frac{1}{3} + 2\frac{7}{8}\right) - \left(5\frac{1}{2} + 1\frac{1}{4} + 2\frac{3}{5}\right)$

2. Evaluate each of the following.

   (a) $\dfrac{2.63 + 5.84 - 0.92}{20.5}$, giving your answer correct to 3 decimal places.

   (b) $(7.8)^2 + \sqrt{22.86}$, giving your answer correct to the nearest whole number.

   (c) $\dfrac{1}{44.5} + 26.81$, giving your answer correct to 1 decimal place.

   (d) $\sqrt{0.0064} + \sqrt[3]{18.1}$, giving your answer correct to 2 decimal places.

   (e) $2.7^3 + \left(\dfrac{1}{12.9}\right)^2$, giving your answer correct to 3 decimal places.

3. With the help of a calculator, find the values of

   (a) $\sqrt{269} - \sqrt{129}$, (b) $\sqrt{269 - 129}$,

   giving your answers correct to 2 decimal places.
   From the answers, what general conclusion can you make about the operations?

Advanced

4. With the help of a calculator, evaluate

   (a) $\sqrt{131} - \sqrt{66} \times 1.3$, correct to 1 decimal place,

   (b) $\sqrt{52.3} - \sqrt[3]{47}$, correct to 2 decimal places.

5. Evaluate the following with the help of a calculator, giving your answers correct to 2 decimal places.

   (a) $\dfrac{4.1 \times 8.92 - 2.97}{42.8}$ (b) $\dfrac{17.1^2 - 18.3^2}{15.9 + 13.2}$ (c) $\dfrac{84.75 + 257.46}{5.9 + 18.2} \times \dfrac{14}{17.5 - 10.91}$

6. Evaluate the following with the help of a calculator, giving your answers correct to 2 decimal places.

   (a) $(12.31 - 7.809)^2$ (b) $\frac{1}{9} + \frac{1}{8}$ (c) $(2.8)^2 + \sqrt{7.68}$ (d) $\dfrac{8.19 + 16.81}{\sqrt{8.29}}$

Test Yourself

1. State whether each of the following statements is true or false.
   (a) $2 < -5$ ( )
   (b) $-5 < -3$ ( )

2. Use a calculator to find the value of $\sqrt{15}$ and from the choices given below, state where this number will lie.
   (a) Between 14 and 16
   (b) Between 3 and 4
   (c) Between 4 and 5
   (d) Between –4 and –3
   (e) Cannot be shown on a number line ( )

3. Which of the expressions below has a value of 6?
   (I) $2^3 - (7 - 5)^2 + 6 \div 3 \times 1$ (II) $-4^3 \div (-2)^3 - 2$
   (III) $24 \div 8 \times 2$ (IV) $21 - 16 + 1$

   (a) I only
   (b) I and II only
   (c) III and IV only
   (d) I, III and IV
   (e) All of the above ( )

4. With the help of a calculator, state whether each of the following statements is true or false.
   (a) 6^3 is greater than $\sqrt[3]{10\,077\,696}$. ( )
   (b) 93^2 is greater than $\sqrt{61\,340\,224}$. ( )
   (c) $16^3 - 4096$ is greater than $\sqrt[3]{-2744}$. ( )
   (d) 2 472 768 is greater than $\sqrt[3]{4096} \times 356^2$. ( )
   (e) $28^3 - 26^3$ is greater than $38^2 + 54^2$. ( )

Go to Workbook Activity 8, page 10

Cumulative Practice 1

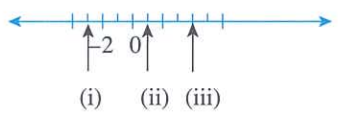
Basic

1. Find the prime factors of the following numbers, leaving your answers in index notation.
   (a) 84 (b) 108 (c) 225

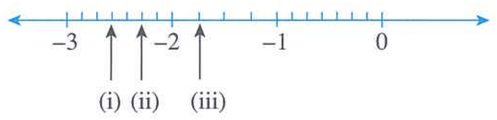
2. Find the HCF of each of the following groups of numbers.
   (a) 28, 64 (b) 54, 84 (c) 108, 120 (d) 42, 112, 70

3. Find the LCM of each of the following groups of numbers.
   (a) 4, 9 (b) 8, 12 (c) 14, 35 (d) 6, 8, 10

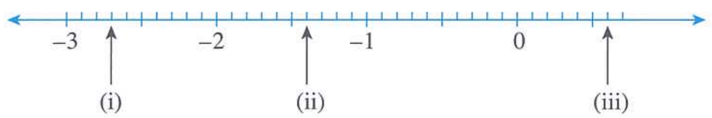
4. (a) What are the integers indicated by the arrows?



   (b) What are the fractions indicated by the arrows?



   (c) What are the decimals indicated by the arrows?



Advanced

5. Put brackets in the following expressions to make them correct.
   (a) $3 \times 4 + 6 - 5 = 25$ (b) $-(-5) + 4 \times 2 - 7 = -45$
   (c) $-15 - 30 \div 10 - 15 = 9$ (d) $4 \times 7 - 9 + 4 = -4$
   (e) $9 - 15 \times 2 - 4 = 12$ (f) $11 + 3 - 15 \div 3 = 9$

6. Use a calculator to evaluate the following.

 (a) $\{[(-4) \times 7] - 4\} \div (-4)$ (b) $\dfrac{3 - 15 + 8}{(-4) \times (-1)}$

 (c) $\dfrac{-12 - (-6) + (-2)}{-5 \times (-4) \times (-1)}$

 (d) $[(-6) \times 3 - 12 \div 2] + [15 \div (-3) \times 4 \times (-2)]$

 (e) $(-18 + 17 - 25 + 10) - [-4 \times (5 - 3) + (-6)]$

 (f) $\{[(-10) \div (8 - 3)] \times (-4)\} \times [(7 - 3) + (8 - 15)]$

7. Find the positive square root of each of the following numbers.

 (a) 784 (b) 441 (c) 1764

 (d) 4356 (e) 6084 (f) 7056

8. What is the length of the side of a square whose area is

 (a) $225\ \text{cm}^2$, (b) $900\ \text{cm}^2$, (c) $1600\ \text{cm}^2$?

 (Hint: Area of a square = $l \times l$ where l = length of the square.)

9. With the aid of a calculator, evaluate

 (a) $[16.02 + (-6.4)] \times 12.218$, (b) $\dfrac{(-11.7)^2 \times 14.82}{3.5^2}$,

 giving your answers correct to 2 decimal places.

10. Use a calculator to find the value of $\sqrt{11\frac{2}{7}}$, giving your answer correct to

 (a) 3 decimal places,

 (b) the nearest whole number.

11. Use a calculator to find the value of $\sqrt{\dfrac{0.736 \times 4.13}{9.52}}$, giving your answer correct to 2 decimal places.

12. Use a calculator to evaluate the following.

 (a) $\sqrt[3]{63} + \sqrt[3]{125}$ (b) $5 \times \sqrt[3]{8}$ (c) $7^2 + \sqrt[3]{64}$

 (d) $\sqrt{26} \times \sqrt{15} - 4$ (e) $\sqrt{81} + \sqrt[3]{729}$ (f) $\sqrt[3]{125} + \sqrt{100} - \sqrt{25}$

 (g) $2^3 + \sqrt{49} - \sqrt[3]{8} + 3^3$

Applications

13. (a) The temperature displayed on a thermometer is 3 °C. What would the reading be if the temperature

  (i) rose by 4 °C, (ii) fell 9 °C from the first reading?

 (b) In each of the following pairs, which reading indicates a warmer temperature?

  (i) 0 °C or –2 °C (ii) –11 °C or –9 °C

14. Three towns, X, Y and Z are 120 m, –25 m and –30 m above sea level respectively. How high is

 (a) town X above town Y, (b) town Y above town Z, (c) town X above town Z?

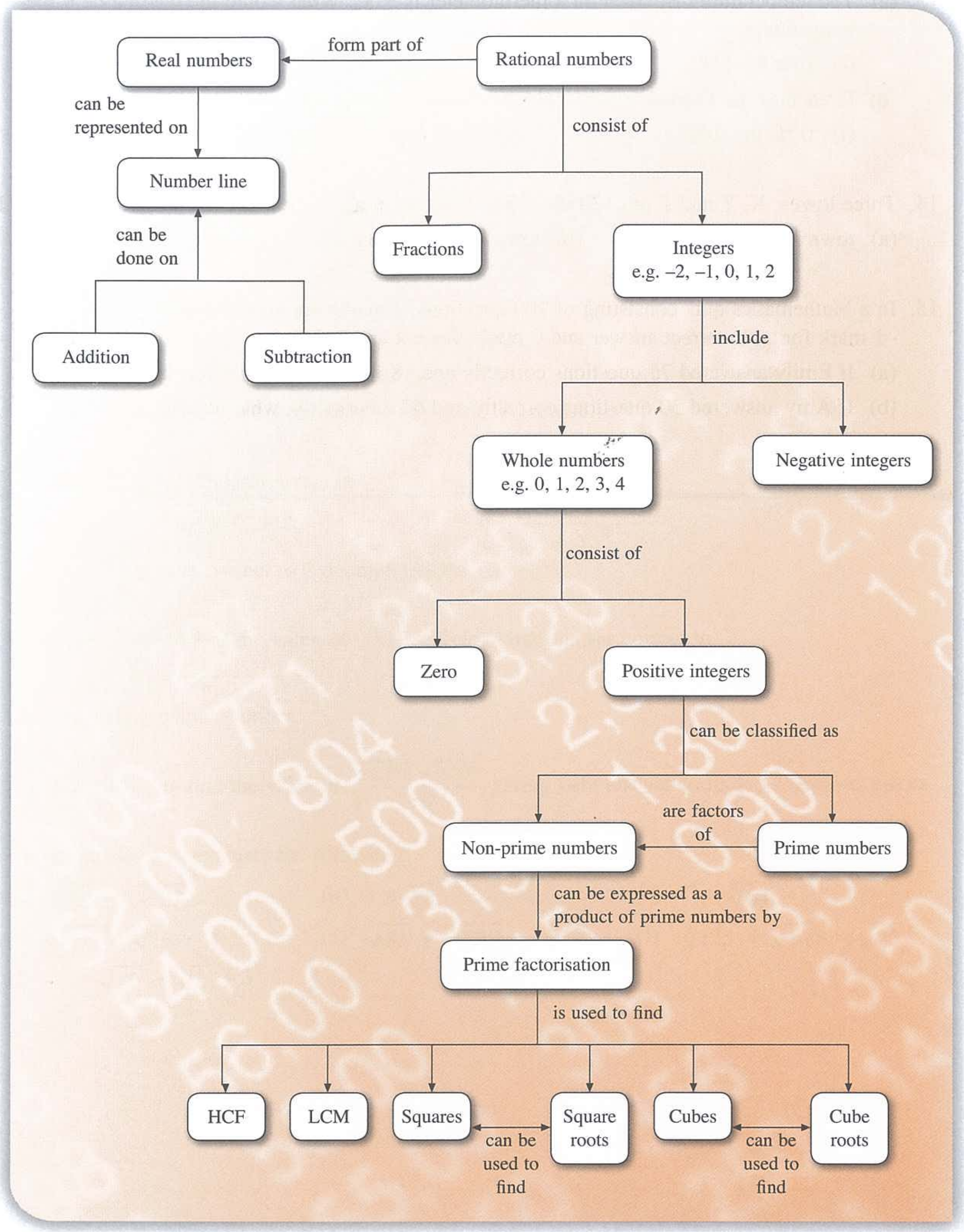
15. In a Mathematics quiz consisting of 100 questions, 2 marks are awarded for a correct answer, –1 mark for an incorrect answer and 0 marks for not answering.

 (a) If Emily answered 75 questions correctly and 18 incorrectly, what was her score?

 (b) If Amy answered 30 questions correctly and 62 incorrectly, what was her score?

Chapter Review

1. The following concept map summarises the scope of coverage in this chapter.



2. Notes

- A **rational** number is a number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

- Operations of integers are as follows.
  - ➢ $+(-a) = -a$
  - ➢ $-(-a) = +a$
  - ➢ $(+a) \times (-b) = -ab$
  - ➢ $(-b) \times (a) = -ab$
  - ➢ $(-a) \times (-b) = ab$
  - ➢ $a \div (-b) = -\frac{a}{b}$
  - ➢ $(-a) \div b = -\frac{a}{b}$
  - ➢ $(-a) \div (-b) = \frac{a}{b}$

- A **prime number** is a number that has exactly two **factors**, **1 and itself**.

Wrap-Up

Go through the checklist below to assess how well you have learnt the topics in this chapter by placing a tick (✓) in the appropriate column.

You are encouraged to review those sections which you need to improve.

Checklist ✓

I understand and am able to	Yes	No	Not sure
1. perform prime factorisation,			
2. find HCF and LCM using prime factorisation,			
3. perform the four operations on integers,			
4. find squares, cubes, square roots and cube roots using prime factorisation,			
5. define rational numbers,			
6. perform the four operations on real numbers,			
7. do long calculations using a scientific calculator,			
8. represent and order numbers on a number line,			
9. use the symbols $<$, $>$, $\leqslant$, $\geqslant$ to order or compare real numbers.			

The difficulties that I encountered in learning numbers are:

The concepts that I can understand and apply are:

CHAPTER 2

Approximation and Estimation

More births in Singapore

It was reported that more than 9000 babies were born in the first quarter of year 2006. This raised hopes of a bumper crop in the nation's population.

Hospitals in the country are reporting more deliveries. There were 2808 births in January, 3066 in February and 3177 in March. In all, there were 9051 babies born in the first quarter which is 407 more than the number for the same period last year.

A spokesman commented that if this trend continued, the final figure for this year might well pass the 40 000 mark.

Singapore needs more clinical-scientists

On 22 April 2006, it was reported that Singapore would need at least 200 clinical-scientists to meet the needs of its fast growing biomedical science industry as well as its desire to be a regional medical hub.

Clinical scientists (ie doctors who treat patients and also do research) are a rare breed all over the world because of the long years of training it takes for one to qualify and the punishing work schedule one has to face.

At the moment, there are fewer than 50 clinical-scientists in the public sector and fewer than 10 graduates pursuing a PhD in medicine at the National University of Singapore (NUS).

The Duke-NUS Graduate Medical School, which opens in year 2007, located at Outram Campus, will start a new course to help nurture clinical-scientists. They will start with an intake of 25 students next year and increase to 50 the following year.

Today there are almost 6000 doctors in active practice with 300 registering each year.

What you see above are the examples in everyday life where approximations are being used. Very often, an approximation is good enough for many situations that involve numbers.

Besides those examples shown above, can you think of other examples where you can find approximations in use?

In this chapter, you will learn to:

- round off numbers to a required number of decimal places,
- find the number of significant figures in a given number,
- round off numbers to a specified number of significant figures,
- estimate the results of computation,
- use estimations in daily life,
- understand truncation errors and rounding off errors.